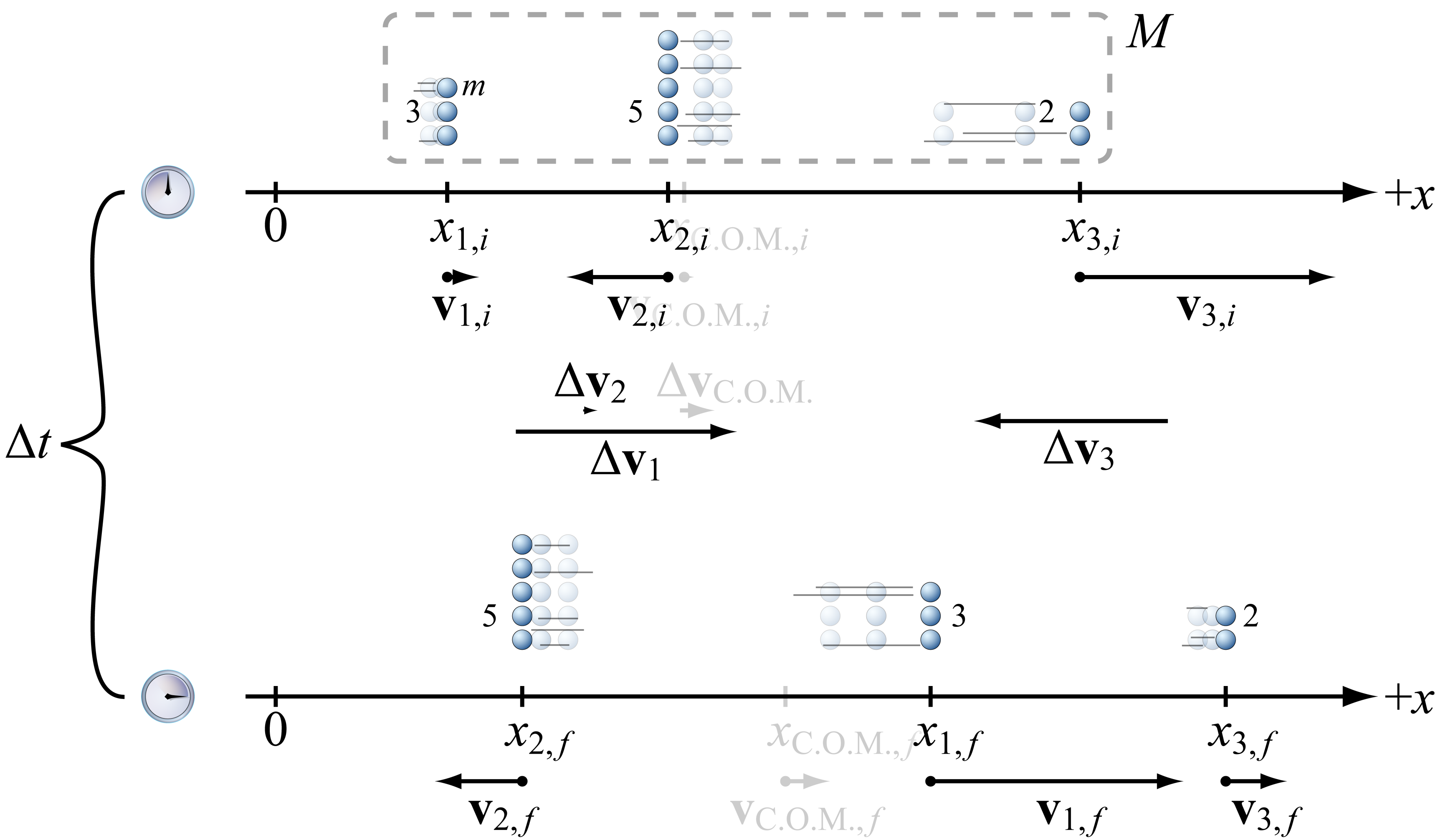
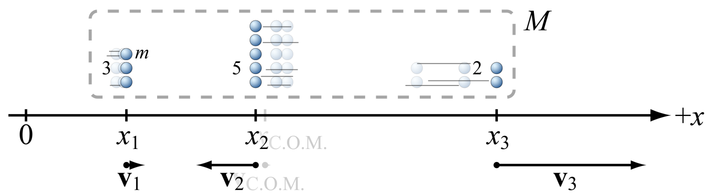
Center of mass



Center of mass

Mass-averaged position



$$x_{\text{C.O.M.}} = \frac{\overbrace{x_1 + x_1 + x_1}^{\text{3 copies}} + \overbrace{x_2 + \cdots + x_2}^{\text{5 copies}} + \overbrace{x_3 + x_3}^{\text{2 copies}}}{10}$$

$$x_{\text{C.O.M.}} = \frac{3}{10}x_1 + \frac{5}{10}x_2 + \frac{2}{10}x_3$$

$$x_{\text{C.O.M.}} := \frac{m_1}{M}x_1 + \frac{m_2}{M}x_2 + \cdots + \frac{m_N}{M}x_N$$

If $\sum_{\text{EXT}\to\text{SYS}} F_x = 0$ and $v_{\text{C.O.M.},x,i} = 0$,

$$x_{\text{C.O.M.},i} = x_{\text{C.O.M.},f}$$

Neatly and graphically represent situation(s)
- Draw before and after situations
- Choose reference position and positive direction

Graphically represent quantities and their relationships
- Specify a marked position on each object (if possible, choose markers that are initially at the same position)

Identify relevant allowed starting point (in)equation(s)
- Relative displacement formula: $\Delta x_{\text{A,C}} = \Delta x_{\text{A,B}} + \Delta x_{\text{B,C}}$
- Constancy of center of mass: $x_{\text{C.O.M.},i} = x_{\text{C.O.M.},f}$

Use numbered steps to show REASoNing
Communicate

Mass-averaged velocity

$$v_{\text{C.O.M.},x,\text{AVG}} = \frac{\Delta x_{\text{C.O.M.}}}{\Delta t} = \frac{x_{\text{C.O.M.},f} - x_{\text{C.O.M.},i}}{\Delta t}$$

$$v_{\text{C.O.M.},x,\text{AVG}} = \frac{\left(\frac{m_1}{M}x_{1,f} + \cdots + \frac{m_N}{M}x_{N,f}\right) - \left(\frac{m_1}{M}x_{1,i} + \cdots + \frac{m_N}{M}x_{N,i}\right)}{\Delta t}$$

$$v_{\text{C.O.M.},x,\text{AVG}} = \frac{m_1}{M}\frac{\Delta x_1}{\Delta t} + \frac{m_2}{M}\frac{\Delta x_2}{\Delta t} + \cdots + \frac{m_N}{M}\frac{\Delta x_N}{\Delta t}$$

$$v_{\text{C.O.M.},x,\text{AVG}} = \frac{m_1}{M}v_{1,x,\text{AVG}} + \frac{m_2}{M}v_{2,x,\text{AVG}} + \cdots + \frac{m_N}{M}v_{N,x,\text{AVG}}$$

is the velocity of a particle with mass and momentum equal to the total mass and total momentum, respectively, of the system

$$v_{\text{C.O.M.},x} = \frac{m_1 v_{1,x} + m_2 v_{2,x} + \cdots + m_N v_{N,x}}{M}$$

$$v_{\text{C.O.M.},x} = \frac{p_{1,x} + p_{2,x} + \cdots + p_{N,x}}{M}$$

$$v_{\text{C.O.M.},x} = \frac{\sum p_x}{M}$$

$$M v_{\text{C.O.M.},x} = \sum p_x$$

In the C.O.M. frame, $v_{\text{C.O.M.},x} = 0$ and $\sum p_x = 0$.

Mass-averaged acceleration

$$a_{\text{C.O.M.},x,\text{AVG}} = \frac{m_1}{M}a_{1,x,\text{AVG}} + \frac{m_2}{M}a_{2,x,\text{AVG}} + \cdots + \frac{m_N}{M}a_{N,x,\text{AVG}}$$

is proportional to net force on system from external sources

$$a_{\text{C.O.M.},x} = \frac{m_1 a_{1,x} + m_2 a_{2,x} + \cdots + m_N a_{N,x}}{M}$$

$$a_{\text{C.O.M.},x} = \frac{\sum_{\text{EXT}\to 1} F_x + \sum_{\text{INT}\to 1} F_x + \cdots + \sum_{\text{EXT}\to N} F_x + \sum_{\text{INT}\to N} F_x}{M}$$

$$a_{\text{C.O.M.},x} = \frac{\sum_{\text{EXT}\to\text{SYS}} F_x}{M}$$